



Preddiplomski sveučilišni studij Arhitektura i Urbanizam

PROGRAM IZ KOLEGIJA
STATIKA

IVAN HORVATIĆ

Ime i prezime studenta

12345

Broj indeksa

ivan.horvatic@gmail.com

e-mail adresa

Potpis

1.6.2017.

Datum predaje

Nositelj kolegija:
Doc.dr.sc. Tanja Kalman Šipoš, dipl.ing.građ.

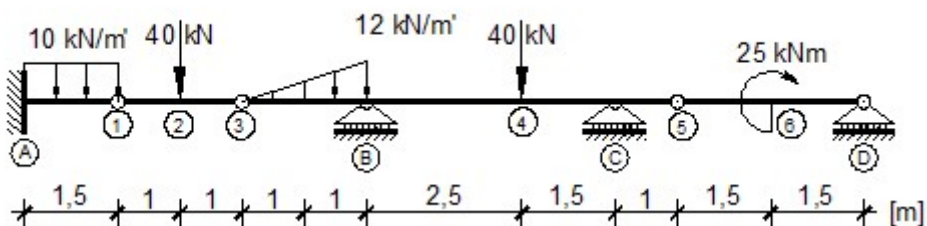
STATIKA

SEMESTRALNI PROGRAM

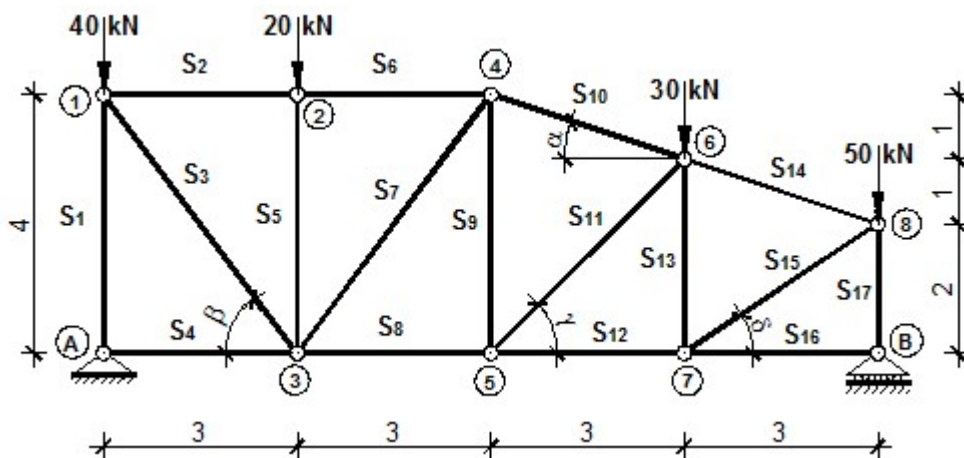
Student: IVAN HORVATIĆ

Akademska godina: 2016/17

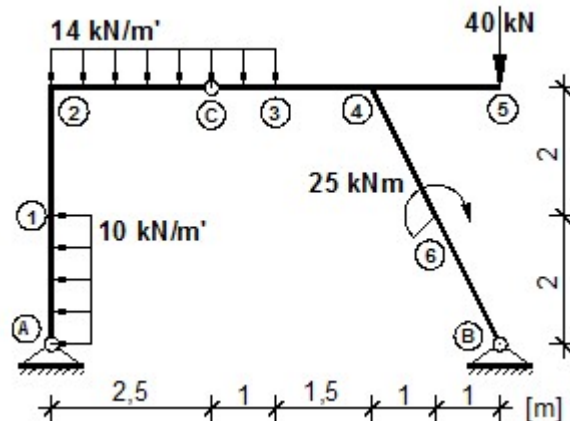
1. Analitičkim postupkom odrediti i nacrtati M i T dijagrame za Gerberov nosač. Za dio 3-B dijagram poprečnih sila izračunati pomoću diferencijalnih odnosa.



2. Za zadani rešetkasti nosač odrediti:
a) reakcije i sile u zadanim čvorovima rešetke A, 1, 2 i 3 pomoću metode čvorova;
b) sile u štapovima S₂, S₃ i S₄ metodom Culmanna, a u štapovima S₁₄, S₁₅ i S₁₆ metodom Rittera.

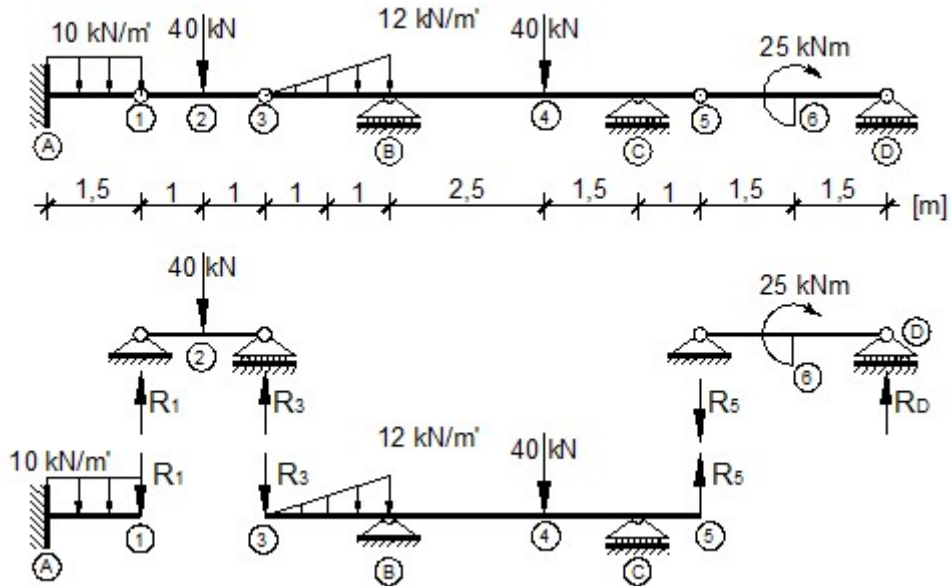


3. Analitičkim postupkom odrediti i nacrtati M, T i N dijagrame za trozglobni okvir.



1. ZADATAK

a) Analitičkim postupkom odrediti i nacrtati M i T dijagrame za Gerberov nosač. Za dio 3-B dijagram poprečnih sila izračunati pomoću diferencijalnih odnosa.

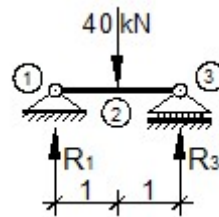


1. Prosta greda 1-3

a) Reakcije:

$$\sum M_1 = 0 \quad + \quad -40 \cdot 1 + R_3 \cdot 2 = 0 \quad R_3 = 20 \text{ kN}$$

$$\sum M_3 = 0 \quad + \quad 40 \cdot 1 - R_1 \cdot 2 = 0 \quad R_1 = 20 \text{ kN}$$



b) Momenti savijanja: $M \left(\begin{array}{c} \curvearrowleft \\ \oplus \\ \curvearrowright \end{array} \right) M$

$$M_1 = M_3 = 0 \text{ kNm}$$

$$M_2 = R_1 \cdot 1 = 20 \text{ kNm}$$

c) Poprečne sile $V \left(\begin{array}{c} \uparrow \\ \oplus \\ \downarrow \end{array} \right) V$

$$V_1 = R_1 = 20 \text{ kN} = V_2^{\text{lijevo}}$$

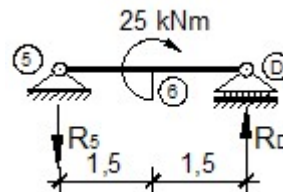
$$V_2^{\text{desno}} = -R_3 = -20 \text{ kN} = V_3$$

2. Prosta greda 6-D

a) Reakcije:

$$\sum M_5 = 0 \quad + \quad -25 + R_D \cdot 3 = 0 \quad R_D = 8,33 \text{ kN}$$

$$\sum M_D = 0 \quad + \quad -25 + R_5 \cdot 3 = 0 \quad R_5 = 8,33 \text{ kN}$$




b) Momenti savijanja: $M \left(\begin{array}{c} \curvearrowleft \\ \oplus \\ \curvearrowright \end{array} \right) M$

$$M_5 = M_D = 0 \text{ kNm}$$

$$M_6^{\text{lijevo}} = -R_6 \cdot 1,5 = -12,5 \text{ kNm}$$

$$M_6^{\text{desno}} = -12,5 + 25 = 12,5 \text{ kNm}$$

c) Poprečne sile 

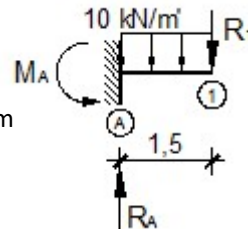
$$V_5 = -R_5 = -8,33 \text{ kN} = V_6 = V_D$$


3. Konzola A-1

a) Reakcije:

$$\sum M_A = 0 \quad \curvearrowright \quad -10 \cdot 1,5 \cdot 0,75 - R_1 \cdot 1,5 + M_A = 0 \quad M_A = 41,25 \text{ kNm}$$


$$\sum F_y = 0 \quad -10 \cdot 1,5 - R_1 + R_A = 0 \quad R_A = 35 \text{ kN}$$



b) Momenti savijanja: 

$$M_A = -41,25 \text{ kNm}$$

$$M_1 = 0 \text{ kNm}$$

c) Poprečne sile 

$$V_A = R_A = 35 \text{ kN}$$

$$V_1 = R_1 = 20 \text{ kN}$$

4. Greda s prepustom 3-5

a) Reakcije:

$$\sum M_B = 0 \quad \curvearrowright$$

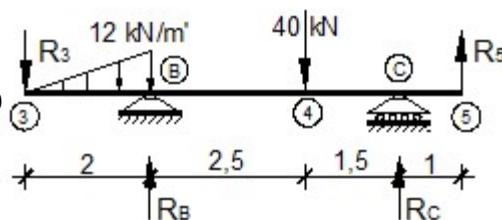
$$R_3 \cdot 2 + \frac{12 \cdot 2}{2} \cdot 0,67 - 40 \cdot 2,5 + R_5 \cdot 5 + R_C \cdot 4 = 0$$


$$R_C = 2,58 \text{ kN}$$

$$\sum M_C = 0 \quad \curvearrowright$$

$$R_3 \cdot 6 + \frac{12 \cdot 2}{2} \cdot 4,67 + 40 \cdot 1,5 + R_5 \cdot 1 - R_B \cdot 4 = 0$$

$$R_B = 61,09 \text{ kN}$$




b) Momenti savijanja: 

$$M_3 = M_5 = 0 \text{ kNm}$$

$$M_B = -R_3 \cdot 2 - \frac{12 \cdot 2}{2} \cdot 0,67 = -48,04 \text{ kNm}$$

$$M_4 = R_5 \cdot 2,5 + R_C \cdot 1,5 = 24,69 \text{ kNm}$$

$$M_C = R_5 \cdot 1 = 8,33 \text{ kNm}$$

c) Poprečne sile 

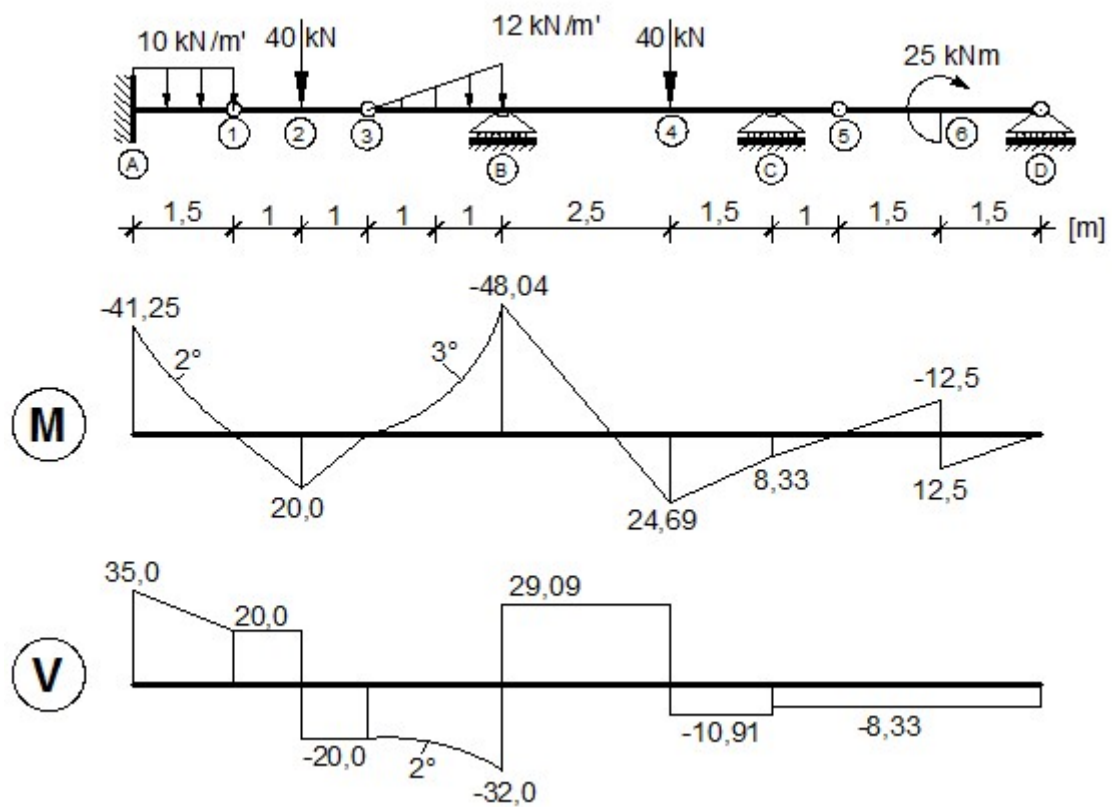
$$V_3 = -R_3 = -20 \text{ kN}$$

$$V_B^{\text{lijevo}} = -R_3 - \frac{12 \cdot 2}{2} = -32 \text{ kN}$$

$$V_B^{\text{desno}} = -32 + R_B = 29,09 \text{ kN} = V_4^{\text{lijevo}}$$

$$V_4^{\text{desno}} = 29,09 - 40 = -10,91 \text{ kN} = V_C^{\text{lijevo}}$$

$$V_C^{\text{desno}} = V_5 = -R_5 = -8,33 \text{ kN}$$



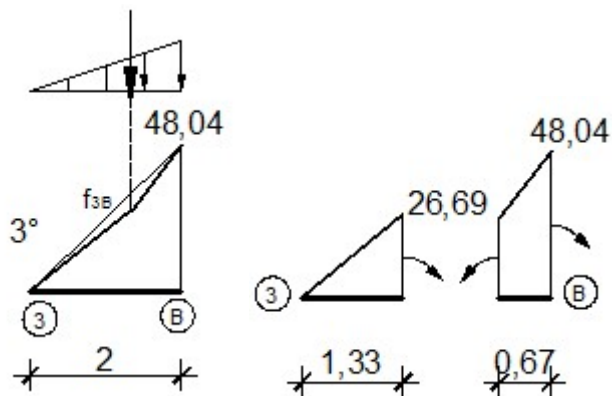
- Određivanje dijagrama poprečnih sila iz poznatog dijagrama momenata

Dio 3-B

$$f_{3B} = 5,33 \text{ kNm}$$

$$M = \frac{2 \cdot 48,04}{3} = 32,03 \text{ kNm}$$

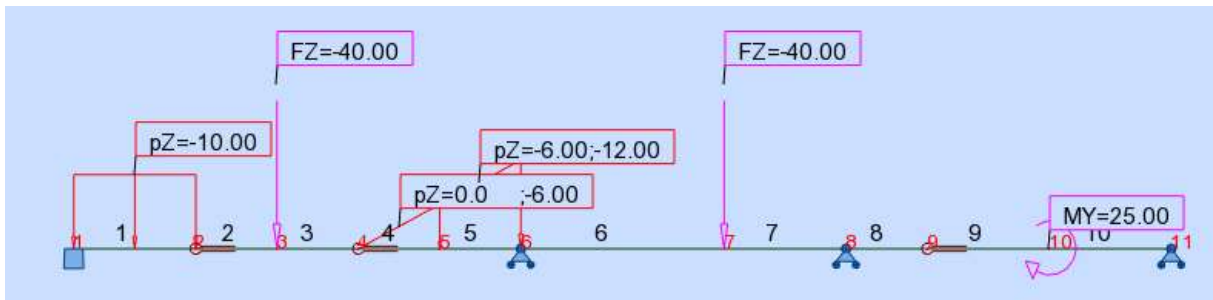
$$M - f_{3B} = 26,69 \text{ kNm}$$



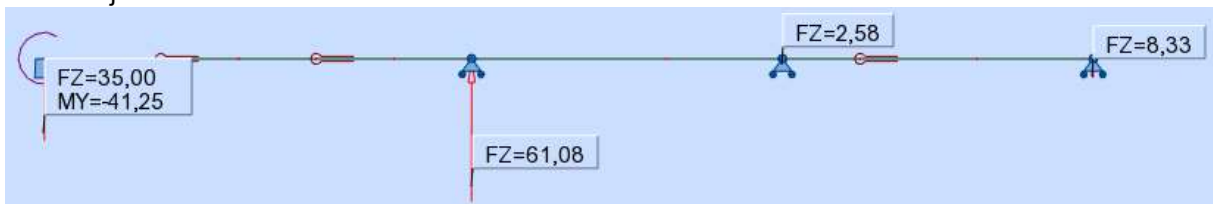
$$V_{3B} = \frac{-26,69}{1,33} = -20,0 \text{ kN}; \quad V_{B3} = \frac{26,69 - 48,04}{0,67} = -32,0 \text{ kN}$$

b) Kontrola proračuna računalom

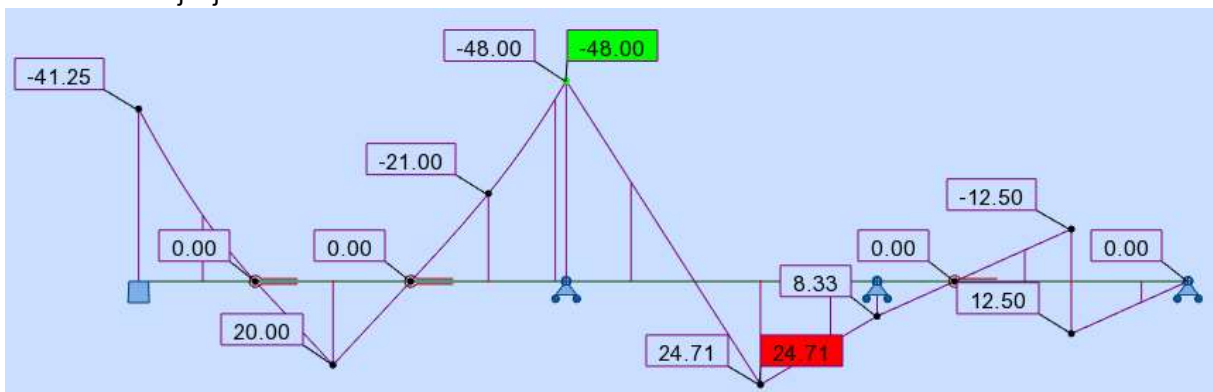
- Numerički model



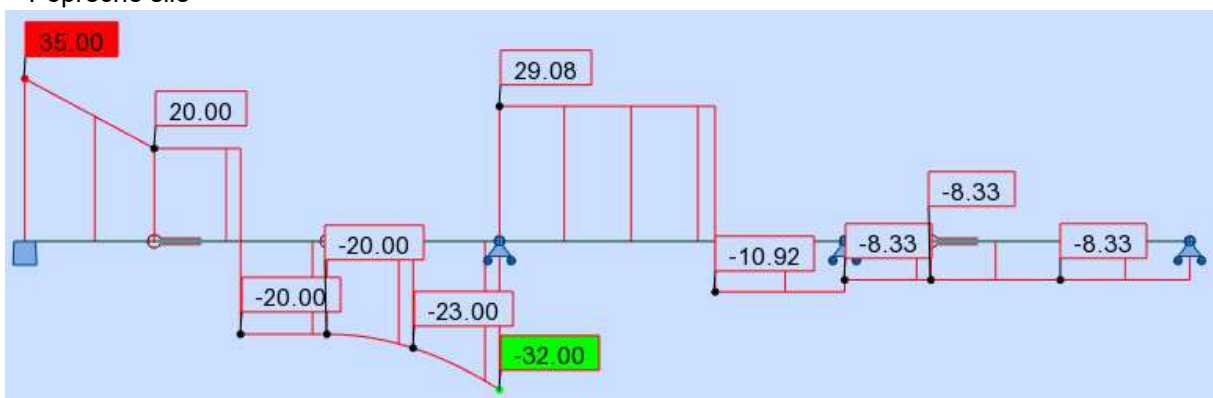
- Reakcije



- Momenti savijanja



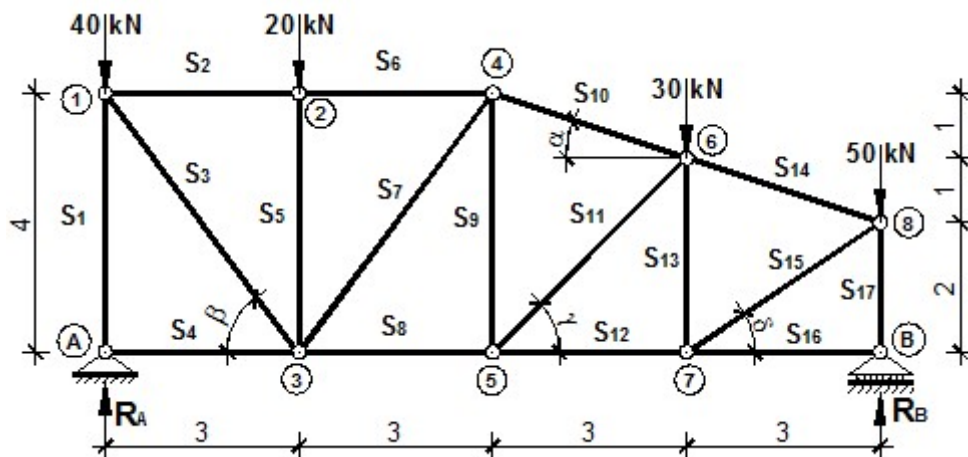
- Poprečne sile



2. ZADATAK

Za zadani rešetkasti nosač odrediti:

a) reakcije i sile u zadanim čvorovima rešetke A, 1, 2 i 3 pomoću metode čvorova;



$$\begin{aligned} \operatorname{tg} \alpha = \frac{1}{3} \Rightarrow \alpha = 18,43^\circ & \quad \operatorname{tg} \beta = \frac{4}{3} \Rightarrow \beta = 53,13^\circ & \quad \operatorname{tg} \gamma = \frac{3}{3} \Rightarrow \gamma = 45^\circ & \quad \operatorname{tg} \delta = \frac{2}{3} \Rightarrow \delta = 33,69^\circ \\ \sin \alpha = 0,32 & \quad \sin \beta = 0,80 & \quad \sin \gamma = 0,71 & \quad \sin \delta = 0,55 \\ \cos \alpha = 0,95 & \quad \cos \beta = 0,60 & \quad \cos \gamma = 0,71 & \quad \cos \delta = 0,83 \end{aligned}$$

1. Reakcije

$$\sum M_A = 0 \quad + \curvearrowright \quad -20 \cdot 3 - 30 \cdot 9 - 50 \cdot 12 + R_B \cdot 12 = 0 \quad R_B = 77,5 \text{ kN}$$

$$\sum M_B = 0 \quad + \curvearrowright \quad 40 \cdot 12 + 20 \cdot 9 + 30 \cdot 3 - R_A \cdot 12 = 0 \quad R_A = 62,5 \text{ kN}$$

Kontrola reakcija:

$$\sum F_y = 0$$

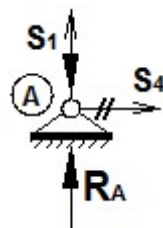
$$R_A + R_B = 40 + 20 + 30 + 50 \\ 140 = 140$$

Čvor A

Napomena: Uvijek pretpostavka vlačnih sila (+) →!

$$\sum F_x = 0 \quad S_4 = 0 \text{ kN}$$

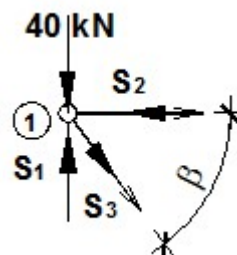
$$\sum F_y = 0 \quad S_1 + R_A = 0 \quad S_1 = -62,5 \text{ kN (tlak)}$$



Čvor 1

$$\sum F_y = 0 \quad S_1 - 40 - S_3 \cdot \sin \beta = 0 \\ S_3 = 28,13 \text{ kN (vlak)}$$

$$\sum F_x = 0 \quad S_2 + S_3 \cdot \cos \beta = 0 \quad S_2 = -16,88 \text{ kN (tlak)}$$

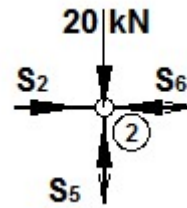


Čvor 2

$$\sum F_y = 0 \quad -S_5 - 20 = 0$$

$$S_5 = -20,0 \text{ kN (tlak)}$$

$$\sum F_x = 0 \quad S_2 + S_6 = 0 \quad S_6 = -16,88 \text{ N (tlak)}$$



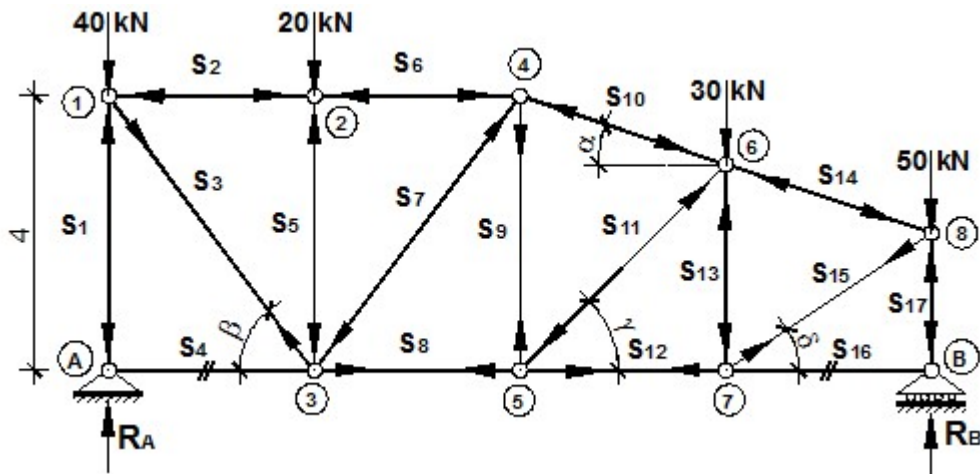
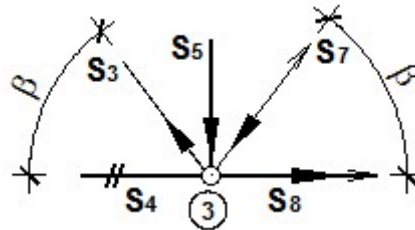
Čvor 3

$$\sum F_y = 0 \quad +S_3 \cdot \sin\beta - S_5 + S_7 \cdot \sin\beta = 0$$

$$S_7 = -3,13 \text{ kN (tlak)}$$

$$\sum F_x = 0 \quad -S_3 \cdot \cos\beta - S_7 \cdot \cos\beta + S_8 = 0$$

$$S_8 = 18,75 \text{ kN (vlak)}$$



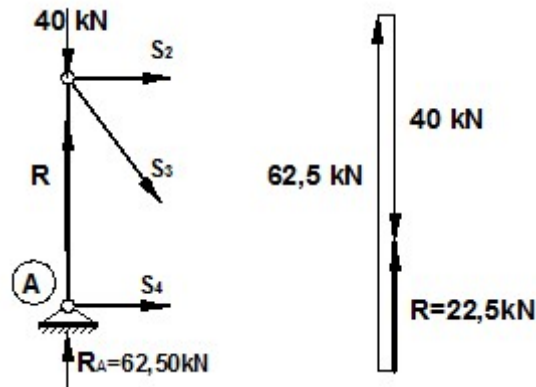
Oznaka	kN	tlak/vlak
S ₁	62,50	tlak
S ₂	16,87	tlak
S ₃	28,12	vlak
S ₄	0	-
S ₅	20,0	tlak
S ₆	16,87	tlak
S ₇	3,12	tlak
S ₈	18,75	vlak

b) sile u štapovima S_2 , S_3 i S_4 metodom Culmanna, a u štapovima S_{14} , S_{15} i S_{16} metodom Rittera.

b1) Metoda Culmanna

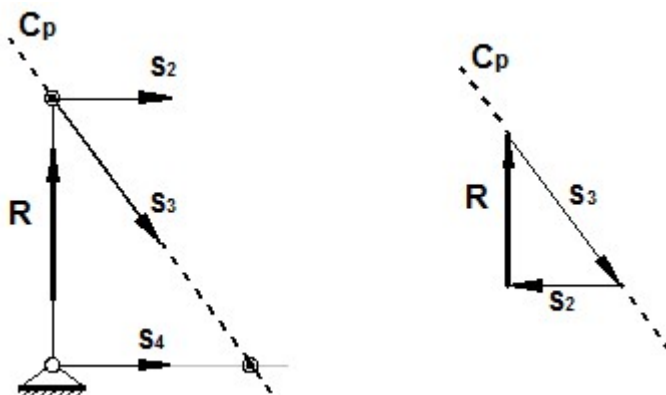
- Položaj rezultante

Za skicirani isječak rešetke (za određivanje sile u presjeku) tražimo vrijednost i položaj rezultante sile pomoću verižnog poligona. U ovom slučaju kada su obje djelujuće sile na jednom istom pravcu nije potrebno raditi verižni poligon jer se rezultanta sile nalazi na pravcu postojećih sila.



- Culmannov pravac

Culmannov pravac određen je dvjema točkama. Prva točka je sjecište rezultante sile i bilo kojeg pravca nepoznatih sila (S_2). Druga točka je mjesto gdje se sijeku preostala dva pravca sile zadanog presjeka (S_3 i S_4). Spajanjem prve i druge točke dobivamo Culmannov pravac C_p .



Očitane vrijednosti sila:

$S_2=16,70\text{kN}$ (tlak);

$S_3=28,05\text{kN}$ (vlak);

$S_4=0\text{kN}$

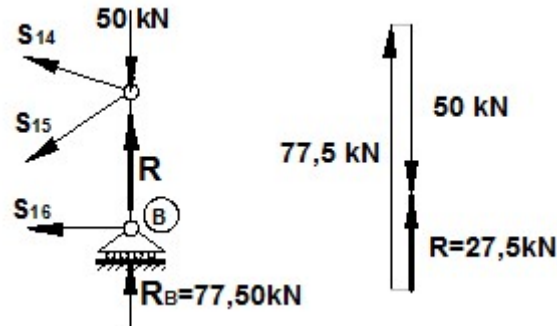
- Određivanje vrijednosti sila u označenim štapovima rešetke

Na Culmannov pravac prvo nanese rezultantu, a zatim silu koja se sječe sa rezultantom u prvoj točki (S_2) i to s iste strane Culmannovog pravca. S druge strane Culmannovog pravca nanose se pravci sile iz druge točke (u ovom slučaju samo pravac sile S_3). Očitavaju se vrijednosti sile i predznaci prema pravilu ravnoteže sile (zatvoren krug sile).

b2) Metoda Rittera

- Položaj rezultante

Za skicirani isječak rešetke (za zadani presjek) tražimo vrijednost i položaj rezultante sila pomoću verižnog poligona. S obzirom da obje sile na isječku djeluju na jednom istom pravcu nije potrebno raditi verižni poligon jer se rezultanta sila nalazi na pravcu postojećih sila.

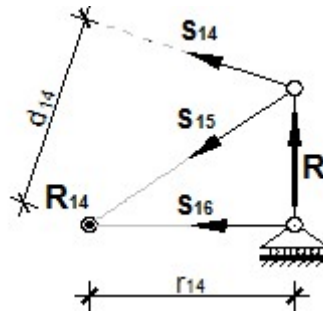


- Položaj ritterovih točaka i određivanje nepoznatih sila u presjeku

Za pravac sile S_{14} ritterova točka se nalazi na sjecištu preostala dva pravca sila u presjeku S_{15} i S_{16} (da bi ostala samo jedna nepoznanica S_{14} koja se rješava pomoću uvjeta ravnoteže $\sum M_{R14}=0$). Zatim se očitava udaljenost d_{14} između ritterove točke i nepoznate sile S_{14} , i udaljenost r_{14} između ritterove točke i rezultante R . Udaljenosti se određuju kao okomice na pravce sila do točke R_{14} .

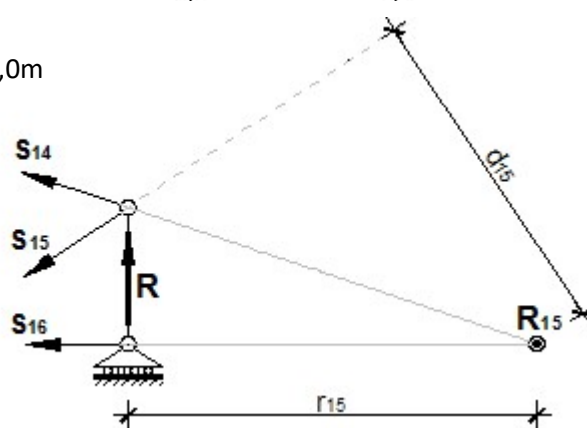
Očitane udaljenosti: $d_{14}=2,85\text{m}$; $r_{14}=3,0\text{m}$

$$\begin{aligned} \sum M_{R14} &= 0 \quad + \\ R \cdot r_{14} + S_{14} \cdot d_{14} &= 0 \\ S_{14} &= -28,95\text{kN} \text{ (tlak)} \end{aligned}$$



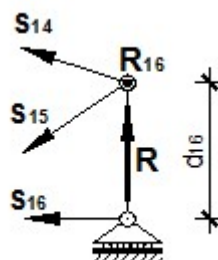
Očitane udaljenosti: $d_{15}=5,0\text{m}$; $r_{15}=6,0\text{m}$

$$\begin{aligned} \sum M_{R15} &= 0 \quad + \\ -R \cdot r_{15} + S_{15} \cdot d_{15} &= 0 \\ S_{15} &= 33,0\text{kN} \text{ (vlak)} \end{aligned}$$



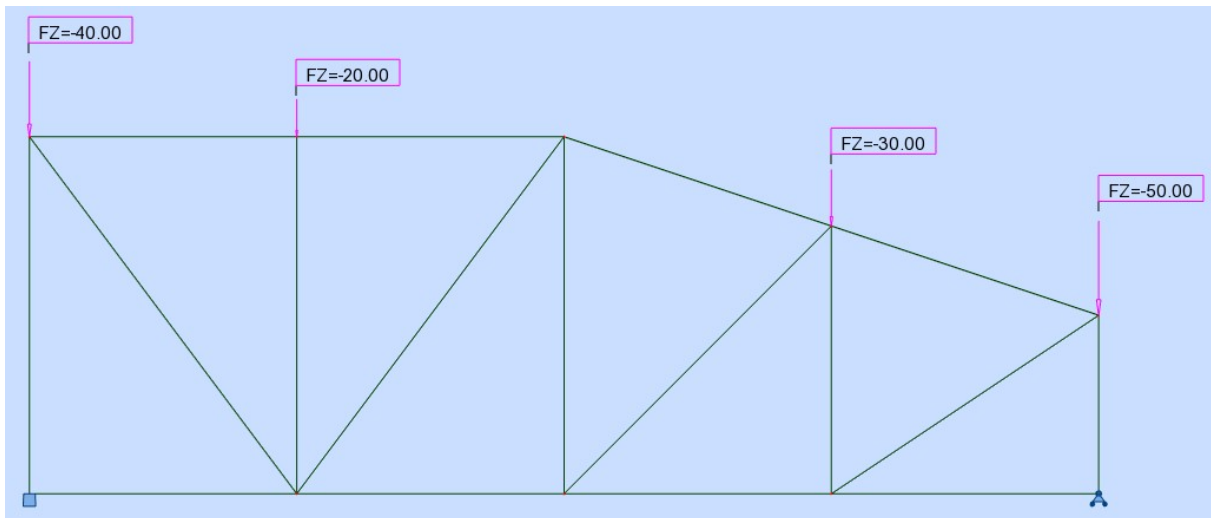
Očitane udaljenosti: $d_{16}=2,0\text{m}$; $r_{16}=0,0\text{m}$

$$\begin{aligned} \sum M_{R16} &= 0 \quad + \\ R \cdot r_{16} - S_{16} \cdot d_{16} &= 0 \\ S_{16} &= 0,0\text{kN} \end{aligned}$$

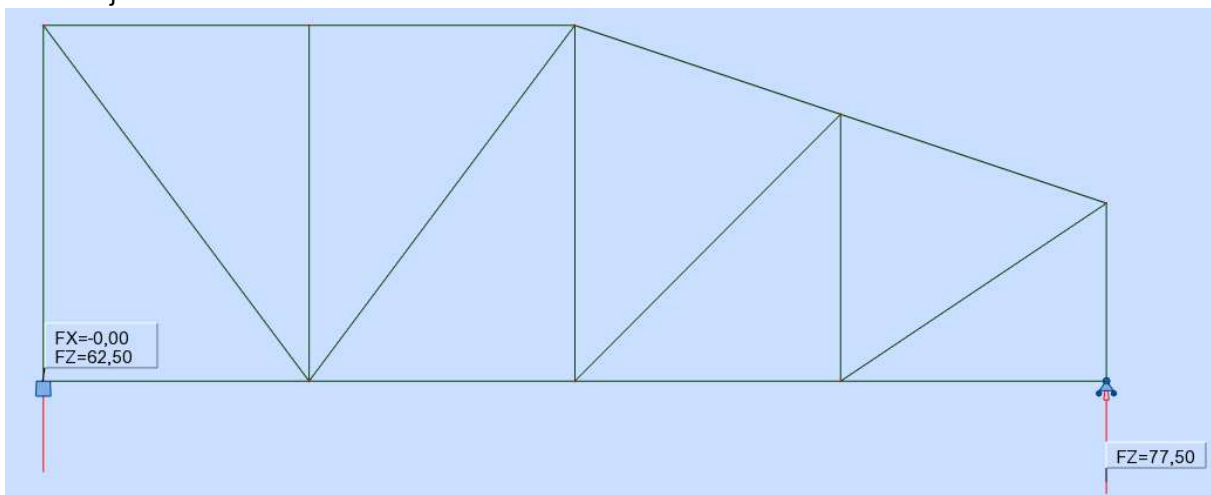


c) Kontrola proračuna računalom

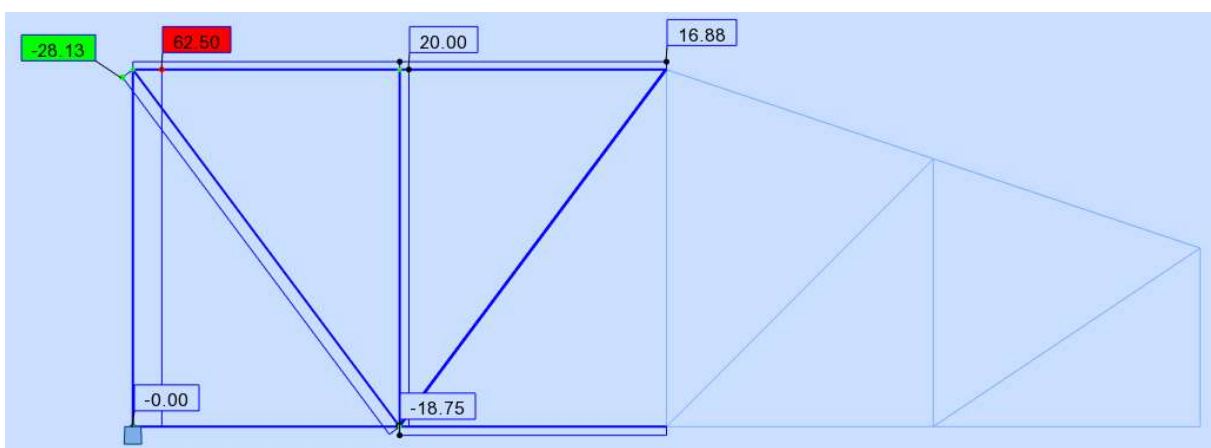
- Numerički model



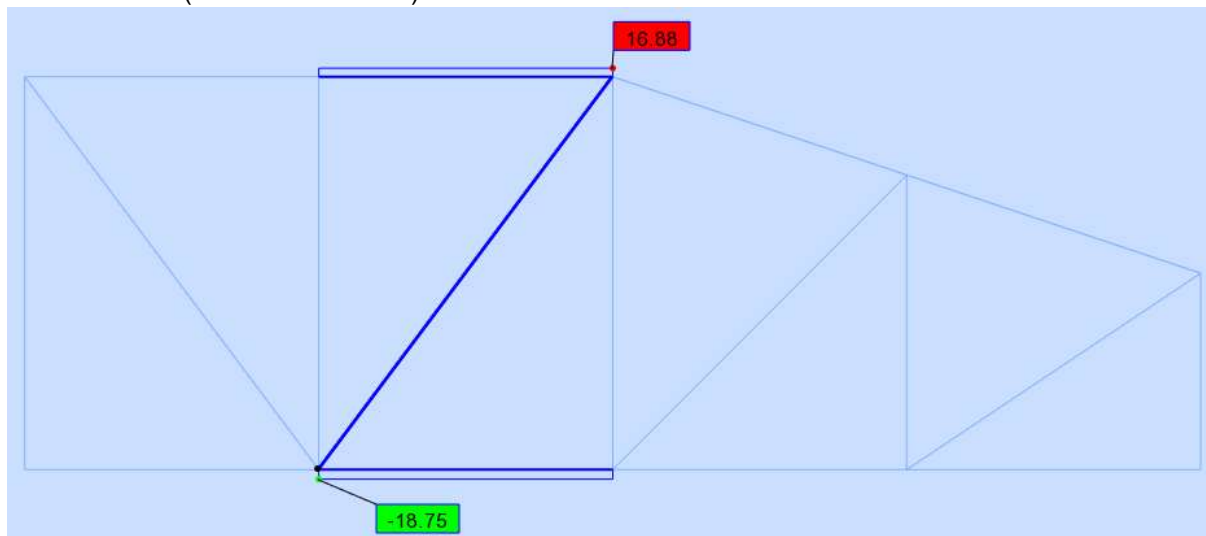
- Reakcije



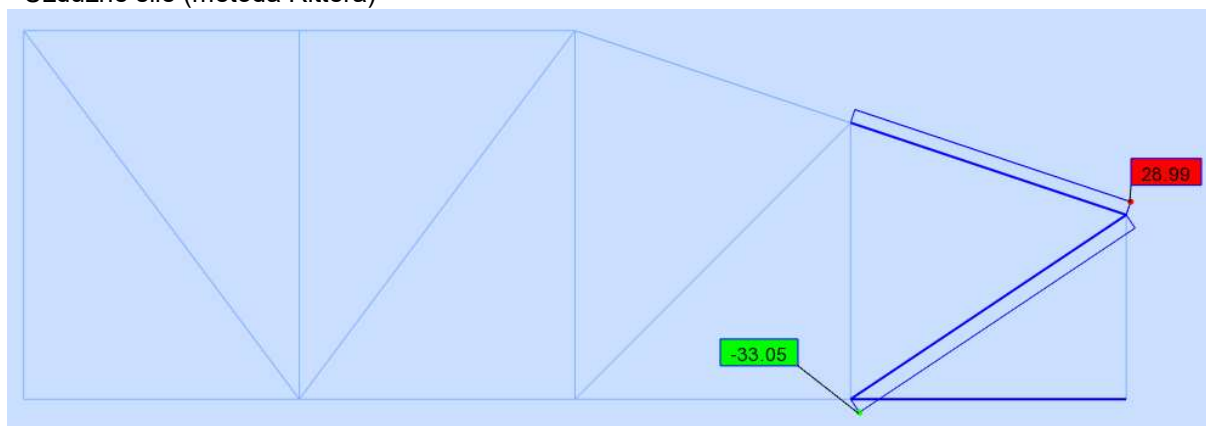
- Uzdužne sile (analitički postupak)



- Uzdužne sile (metoda Culmanna)

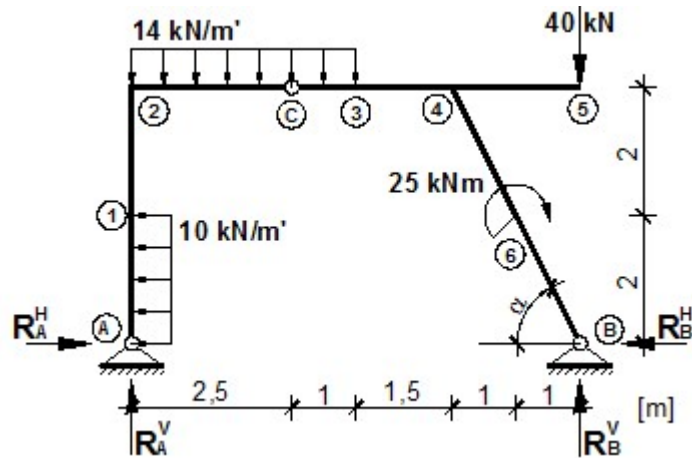


- Uzdužne sile (metoda Rittera)



3. ZADATAK

a) Analitičkim postupkom odrediti i nacrtati M, T i N dijagrame za trozglobni okvir.



$$\begin{aligned} \operatorname{tg} \alpha &= \frac{4}{2} \Rightarrow \alpha = 63,43^\circ \\ \sin \alpha &= 0,89 \\ \cos \alpha &= 0,45 \end{aligned}$$

1. Reakcije

$$\sum M_A = 0 \quad + \curvearrowright$$

$$10 \cdot 2 \cdot 1 - 14 \cdot 3,5 \cdot 1,75 - 25 - 40 \cdot 7 + R_B^V \cdot 7 = 0$$

$$R_B^V = 52,86 \text{ kN}$$

$$\sum M_B = 0 \quad + \curvearrowright$$

$$10 \cdot 2 \cdot 1 + 14 \cdot 3,5 \cdot 5,25 - 25 - R_A^V \cdot 7 = 0$$

$$R_A^V = 36,04 \text{ kN}$$

$$\sum M_C^{\text{desno}} = 0 \quad + \curvearrowright \quad -14 \cdot 1 \cdot 0,5 - 40 \cdot 4,5 - 25 + R_B^V \cdot 4,5 - R_B^H \cdot 4 = 0 \quad R_B^H = 6,58 \text{ kN}$$

$$\sum M_C^{\text{lijevo}} = 0 \quad + \curvearrowright$$

$$14 \cdot 2,5 \cdot 1,25 - 10 \cdot 2 \cdot 3 - R_A^V \cdot 2,5 - R_A^H \cdot 4 = 0$$

$$R_A^H = 26,58 \text{ kN}$$

Kontrola reakcija:

$$\sum F_x = 0$$

$$\begin{aligned} R_A^H &= R_B^H + 10 \cdot 2 \\ 26,58 &= 6,58 \end{aligned}$$

$$\sum F_y = 0$$

$$\begin{aligned} R_A^V + R_B^V &= 40 + 14 \cdot 3,5 \\ 89,0 &= 89,0 \end{aligned}$$

2. Unutarnje sile

a) Momenti savijanja M

$$M_A = M_B = M_C = M_5 = 0 \text{ kNm}$$

$$M_1 = 10 \cdot 2 \cdot 1 - R_A^H \cdot 2 = -33,16 \text{ kNm}$$

$$M_2 = -10 \cdot 2 \cdot 3 + R_A^H \cdot 4 = -46,32 \text{ kNm}$$

$$M_3 = R_A^V \cdot 3,5 - R_A^H \cdot 4 + 10 \cdot 2 \cdot 3 - 14 \cdot 3,5 \cdot 1,75 = -5,93 \text{ kNm}$$

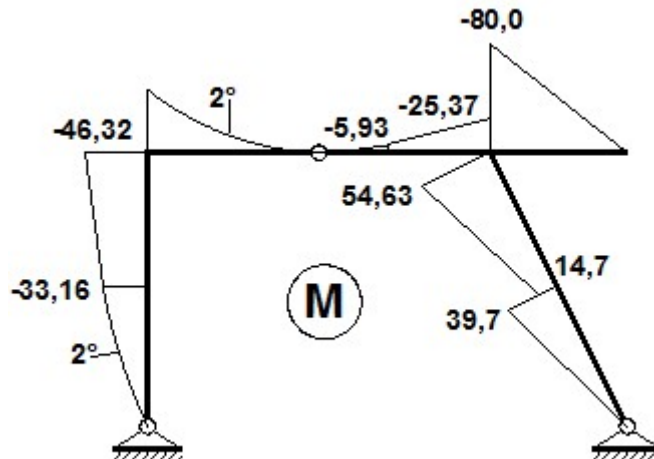
$$M_4^{\text{lijevo}} = R_A^V \cdot 5 - R_A^H \cdot 4 + 10 \cdot 2 \cdot 3 - 14 \cdot 3,5 \cdot 3,25 = -25,37 \text{ kNm}$$

$$M_4^{\text{desno}} = -40 \cdot 2 = -80 \text{ kNm}$$

$$M_4^{\text{dolje}} = 80,0 - 25,37 = 54,63 \text{ kNm}$$

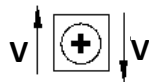
$$M_6^{\text{dolje}} = R_B^V \cdot 1 - R_B^H \cdot 2 = 39,7 \text{ kNm}$$

$$M_6^{\text{gore}} = 39,7 - 25 = 14,7 \text{ kNm}$$



b) Poprečne sile

$V=T$



$$V_A = -R_A^H = -26,58 \text{ kN}$$

$$V_1 = -26,58 + 10 \cdot 2 = -6,58 \text{ kN} = V_2^{\text{dolje}}$$

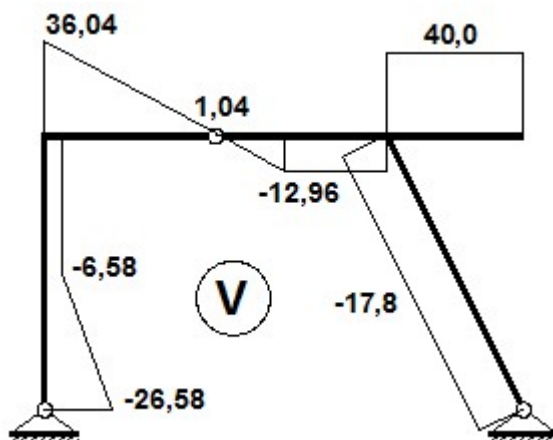
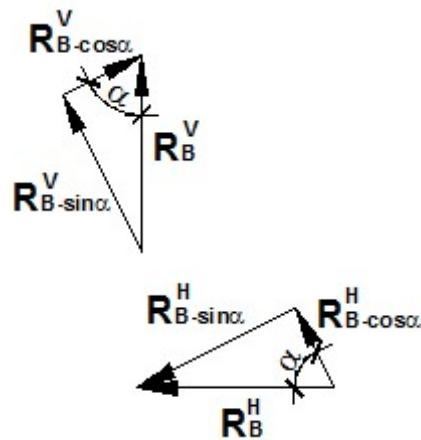
$$V_2^{\text{desno}} = R_A^V = 36,04 \text{ kN}$$

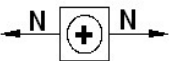
$$V_C = 36,04 - 14 \cdot 2,5 = 1,04 \text{ kN}$$

$$V_3 = 36,04 - 14 \cdot 3,5 = -12,96 \text{ kN} = V_4^{\text{lijevo}}$$

$$V_5 = V_4^{\text{desno}} = 40 \text{ kN}$$

$$V_4^{\text{dolje}} = R_B^H \cdot \sin \alpha - R_B^V \cdot \cos \alpha = -17,80 \text{ kN} = V_B = V_5$$



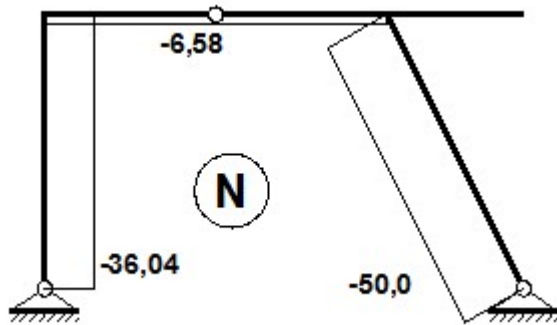
c) Uzdužne sile 

$$N_A = -R_A^V = -36,04 \text{ kN} = N_1 = N_2^{\text{dolje}}$$

$$N_2^{\text{desno}} = -R_A^H + 10 \cdot 2 = -6,58 \text{ kN} = N_C = N_3 = N_4^{\text{lijevo}}$$

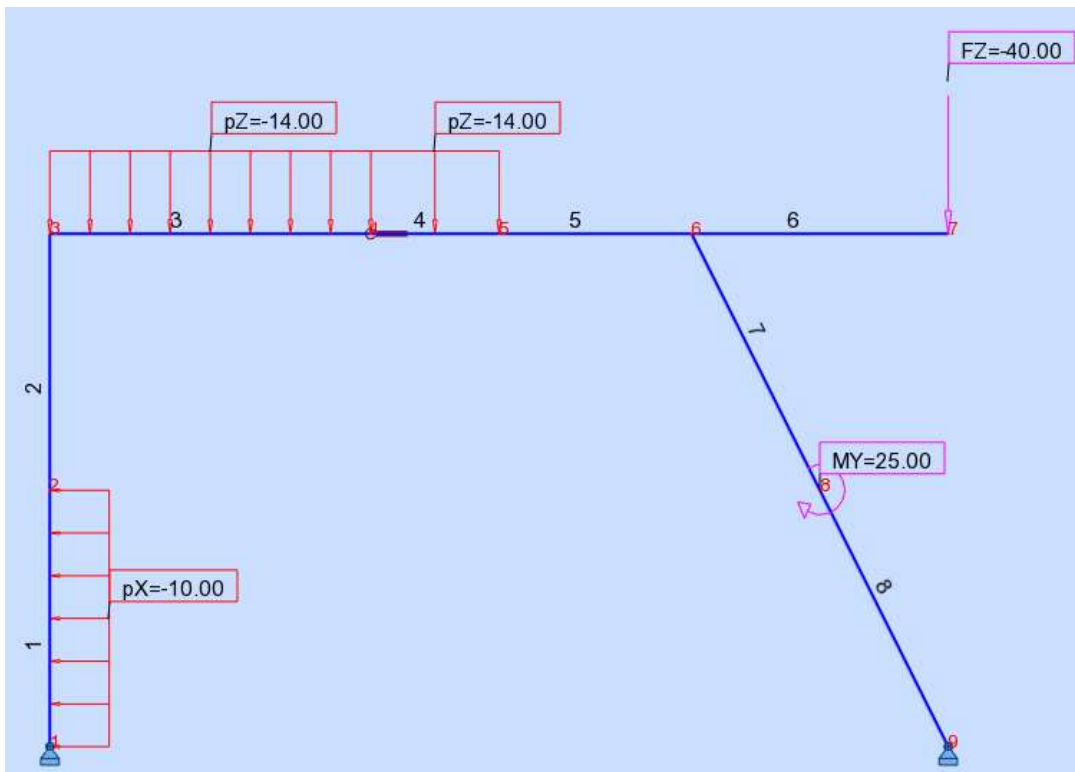
$$N_5 = 0 \text{ kN} = N_4^{\text{lijevo}}$$

$$N_4^{\text{dolje}} = -R_B^V \cdot \sin \alpha - R_B^H \cdot \cos \alpha = -50,0 \text{ kN} = N_5 = N_B$$

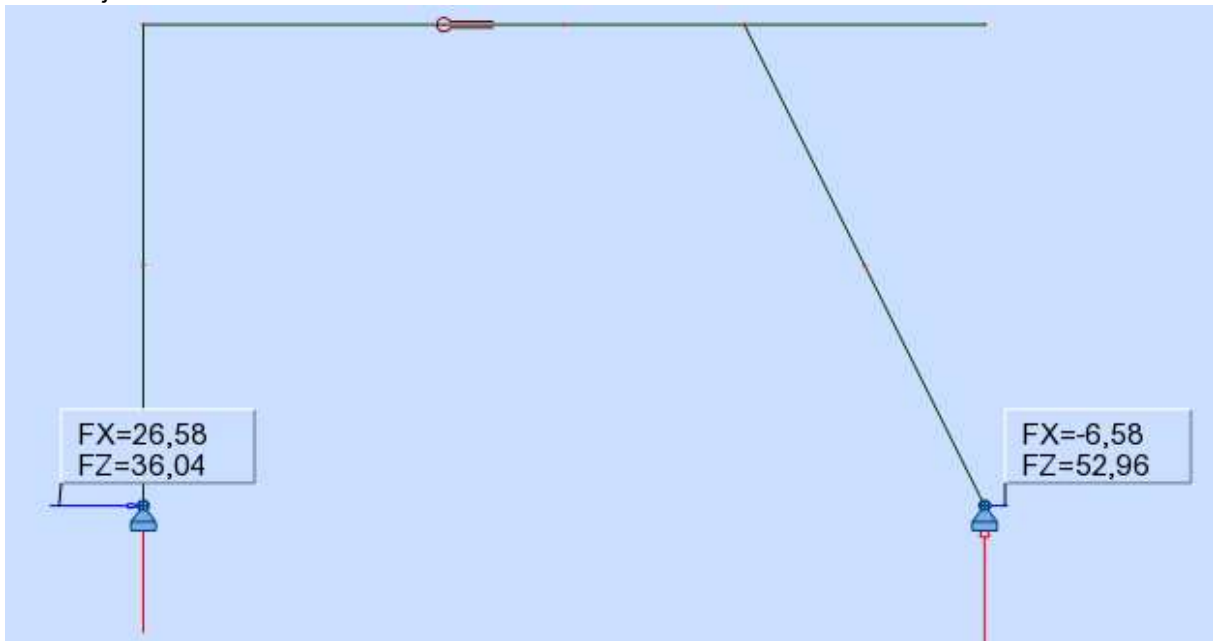


b) Kontrola proračuna računalom

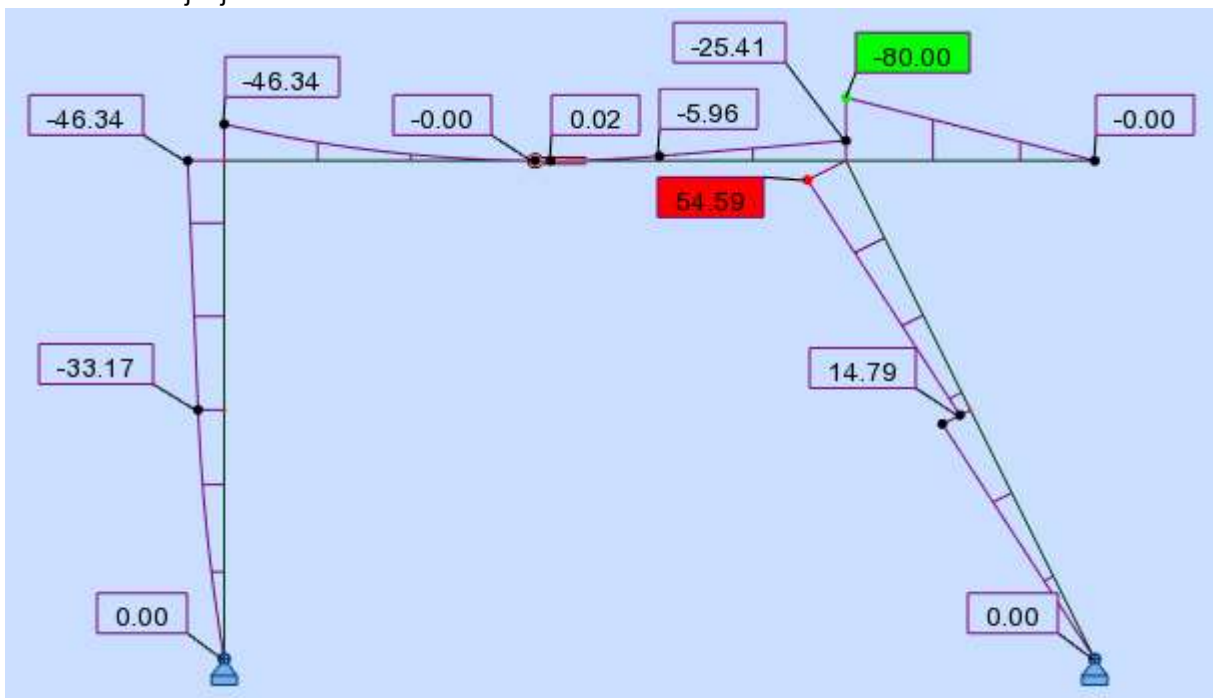
- Numerički model



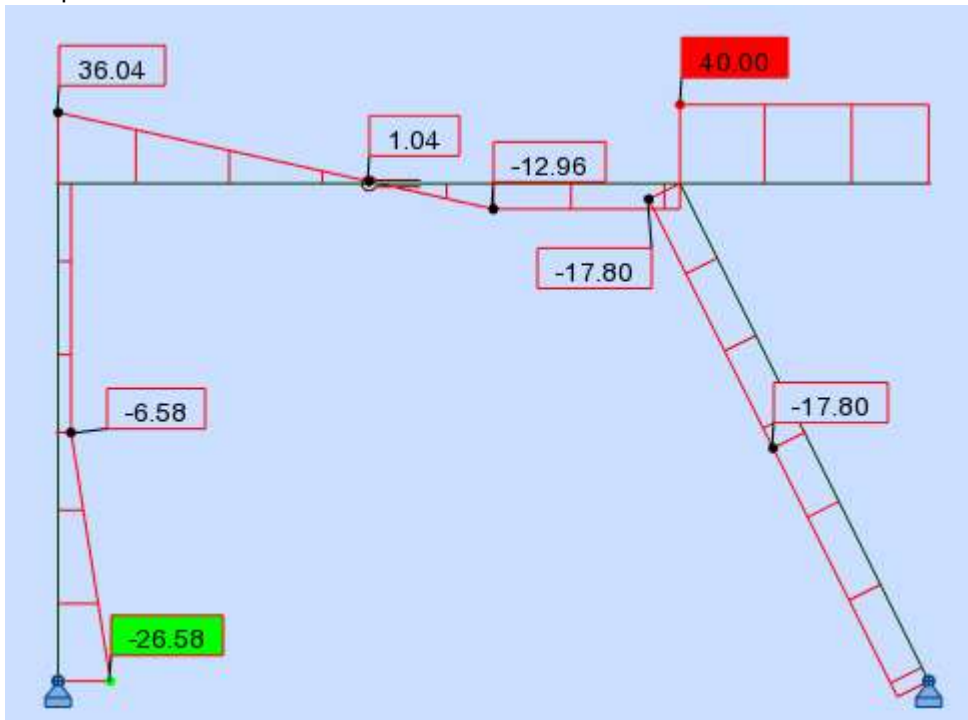
- Reakcije



- Momenti savijanja



- Poprečne sile



- Uzdužne sile

